

IX. *Short and easy Methods for finding*
 (1.) *The Quantity of Time contained in any given Number of Mean Lunations ;* (2.) *The Number of Mean Lunations contained in any given Quantity of Time ;* (3.) *The Number of Troy Pounds contained in any given Number of Avoirdupoise Pounds, and vice versa ;* (4.) *The Quantity and Weight of Water contained in a full Pipe of any given Height, and Diameter of Bore ; and consequently, to find what Degree of Power would be required to work a common Pump, or any other Hydraulic Engine, when the Diameter of the Pump-bore, and the Height to which the Water is to be raised therein, are given. Communicated by Mr. James Ferguson, F. R. S.*

Read Feb. 7, 1765. **S**OME time ago, just as I was finishing a table for shewing the quantity of time contained in any given number of mean lunations within the compass of 6000 Julian years, intending thereby to examine how near our present astronomical tables would shew the times of some antient eclipses which we have on record, when the proper equations of the solar and lunar motions were
 I taken

taken into the account; I was visited by my friend William Rivet, Esq; of the Inner Temple, who said he wished he had come sooner, because he could have put me upon a much shorter method of computation. I desired him to shew me his method, which he did most readily and chearfully. “ It was
 “ only to reduce the odd hours, minutes, seconds,
 “ and thirds, &c. above the integral days of a luna-
 “ tion, into the decimal parts of a day; which num-
 “ be of days and decimal parts, being nine times
 “ added together, will be equal to the time contained
 “ in nine mean lunations. And from these, the
 “ time contained in any other assigned number may
 “ be found, as follows.

A Table shewing the quantity of time contained in any given number of mean lunations. The mean lunation being 29 days 12^h 44' 3" 2''' 58''''; or 29.53059085108 days.

Lun.	Days.	Decimals of a Day.
1	29.	53059085108
2	59.	06118170216
3	88.	59177255324
4	118.	12236340432
5	147.	65295425540
6	177.	18354510648
7	206.	71413595756
8	236.	24472680864
9	265.	77531765972

EXPLANATION.

For tens of lunations, remove the decimal point one place forward; for hundreds of lunations, two places; for thousands, three places; and so on, as in the annexed example; and then the remaining decimals may be reduced into hours, minutes, seconds, &c. by the common method of reducing decimals to the known parts of an integer.

EXAMPLE.

EXAMPLE.

In 74212 mean lunations. Qu. How many days, hours, minutes, and seconds?

Lun.	Days	Decim. of a Day.
70000	2067141.	3595756
4000	118122.	36340432
200	5906.	118170216
10	295.305908	5108
2	59.06118170216	
74212	2191524.	20824034896

Anf. 2191524 days and .20824034896 decimal parts of a day.

And by reduction, 2191524 days contain 6000 Julian years, 24 days, and .20824034896 decimal parts of a day, contain 4 hours, 59 minutes, 51 seconds, $57 \frac{9}{10000}$ thirds. But in practice, it is sufficient to take in four or five of the decimal figures.

Having got this hint from Mr. Rivet, I reversed it into a way of finding the number of mean lunations contained in any given quantity of time; for which purpose I calculated the following table, upon the above length of a mean lunation; which comes the nearest to the truth of any length I have yet found, when carried back from the present times to the recorded times of antient eclipses, if the proper equations depending upon the anomalies of the sun and moon are applied to the mean times of new and full moons.

A Table

A Table shewing the number of mean lunations contained in any given quantity of time.

Y.	Lun.	Decimals of a Lunation.	H.	L.	Decimals of a Lunation.	S.	L.	Decimals of a Lunation.
1		12.36853003863	1		0.00141096624	1		1.0000009193
2		24.73706007726	2		0.00282193248	2		2.00000078387
3		37.10559011589	3		0.00423289872	3		3.00000117580
4		49.47412015451	4		0.00564386496	4		4.00000156774
5		61.84265019314	5		0.00705483120	5		5.00000195967
6		74.21118023176	6		0.00846579744	6		6.00000235161
7		86.57971027039	7		0.00987676368	7		7.00000274354
8		98.94824030902	8		0.01128772992	8		8.00000313548
9		111.31677034765	9		0.01269869616	9		9.00000352741

D.	L.	Decimals of a Lunation.	M.	L.	Decimals of a Lunation.	Th.	L.	Decimals of a Lunation.
1		0.0338631897600	1		0.00002351610	1		0.00000000653
2		0.0677263795200	2		0.00004703221	2		0.00000001306
3		0.1015895692800	3		0.00007054831	3		0.00000001959
4		0.1354527590400	4		0.00009406442	4		0.00000002613
5		0.1693159488000	5		0.00011758052	5		0.00000003266
6		0.2031791385600	6		0.00014109662	6		0.00000003919
7		0.2370423283200	7		0.00016461273	7		0.00000004573
8		0.2709055180800	8		0.00018812883	8		0.00000005226
9		0.3047687078400	9		0.00021164494	9		0.00000005879

EXPLANATION.

For tens of years, days, hours, minutes, or seconds, remove the decimal point one place forward; for hundreds, two places; for thousands, three places; and so on, as in the annexed example, which is the converse of the former one.

EXAMPLE.

EXAMPLE.

In 6000 Julian years, 24 days, 4 hours, 59 minutes, 52 seconds, *Qu.* How many mean lunations?

		Lun. Dec. of a L.
Years	6000	74211.18023176
Days	20	— 0.677263795
Hours	4	— 0.135452759
Minutes	4	— 0.005643865
Seconds	50	— 0.001175805
	9	— 0.000211645
	50	— 0.000019596
	2	— 0.000000784
Answer		74212.00000009

This short method may be useful in many other cases; but, as yet, I have only applied it to two. The first of which is, to find the number of troy pounds contained in any given number of avoirdupoise pounds, and the reverse. The second is to find the quantity and weight of water that would fill an upright pipe of any given diameter and height: and consequently, to know what power would be required to work a common pump, or any other hydraulic engine, when the diameter of the bore of the pump, and the height to which the water is to be raised, are given; proper allowance being made for friction. These two cases are as follows.

Troy weight compared with avoirdupoise weight.

One troy pound contains 5760 grains ; and
 One avoirdupoise pound contains 7000 grains.

Therefore,

175 troy pounds are equal to 144 avoird. pounds ; and
 175 troy ounces are equal to 192 avoird. ounces.

On these principles, the two following Tables are constructed.

1. A Table shewing the number of avoirdup. pounds contained in any given number of troy pounds.

2. A Table shewing the number of troy pounds contained in any given number of avoirdupoise pounds.

Troy Pounds	Avoirdupoise pounds and decimals of a pound.
1	0.822857142857143
2	1.645714285714286
3	2.468571428571429
4	3.291428571428572
5	4.114285714285715
6	4.937142857142857
7	5.760000000000000
8	6.582857142857143
9	7.405714285714286

Avoird. Pounds	Troy pounds and decimals of a pound.
1	1.215277777777778
2	2.430555555555556
3	3.645833333333333
4	4.861111111111111
5	6.076388888888889
6	7.291666666666667
7	8.506944444444444
8	9.722222222222222
9	10.937500000000000

EXPLANATION.

For tens of pounds, remove the decimal point one place forward ; for hundreds of pounds, two places ; for thousands, three places ; and so on, as in the following examples.

EXAMPLE

EXAMPLE I.

In 175 troy pounds, *Qy.* How many avoirdupoise pounds?

Troy	Avoirdupoise
100	82.2857142857143
70	57.6000000000000
5	4.1142857142857
175	144.0000000000000

Answer, 144.

EXAMPLE 2.

In 144 avoirdupoise pounds, *Qy.* How many troy pounds?

Av.	Troy
100	121.5277777777778
40	48.6111111111111
4	4.8611111111111
144	175.0000000000000

Answer, 175.

When any significant decimal figures remain in the sum, after integral pounds, they are easily reduced into the known parts of a pound: seeing that, in troy weight, 24 grains make one penny-weight, 20 penny-weights make an ounce, and 12

ounces make a pound. And, in avoirdupoise weight, 16 drams make an ounce, and 16 ounces make a pound.

A Table by which the quantity and weight of water in a cylindrical pipe of any given diameter and perpendicular height may be found.

Diameter of the cylindrical bore one inch.

Feet high.	Quan. of water in cubic inches.	Weight in troy ounces.	In avoirdupoise ounces.
1	9.4247781	4.9712340	5.4541539
2	18.8495562	9.9424680	10.9013078
3	28.2743343	14.9137020	16.3624617
4	37.6991124	19.8849360	21.8166156
5	47.1238905	24.8561700	27.2707695
6	56.5486686	29.8274040	32.7249234
7	65.9734467	34.7986380	38.1790773
8	75.3982248	39.7698720	43.6332312
9	84.8230029	44.7411060	49.0873851

EXPLANATION.

For tens of feet high, remove the decimal point one place forward; for hundreds of feet, two places, for thousands of feet, three places, and so on: and you will have the quantity and weight of water in the cylindrical pipe, supposing its diameter to be one inch.

The contents of cylinders of equal heights are to one another as the squares of their diameters. Therefore, having found the quantity and weight of water in a cylinder of one inch bore, you may find the same for any other larger bore, by multiplying the above-found quantities by the square of the diameter of

of the given bore, taken in inches. Thus, suppose the given bore was 10 inches in diameter (the square of which is 100) and the height of water in the pipe of 10 inches bore was 85 feet, *Qu.* The quantity and weight of the water ?

F. high.	Cubic inches.	Troy ounces.	Avoird. ounces.
80	753.982248	397.698720	436.332312
5	47 123890	24.856170	27.270769
85	801.106138	422.554890	493.603081
Mult. by	100	100	100
Ans ^w .	80110.613800	42255.489000	46360.308100

Which number (80110.6) of cubic inches being divided by 231, gives 346.8 for the number of wine gallons ; and the respective weights (42255.489 troy ounces, and 46360.308 avoirdupoise ounces) being divided, the first by 12, and the last by 16, give 3521.29 for the number of troy pounds, and 2897.513 for the number of avoirdupoise pounds. And so much would be required to balance this quantity of water in a pump, or any other hydraulic engine, and as much more to raise it as would be sufficient to overcome the friction of the engine.

The reason for removing the decimal points one place forward for tens, two places for hundreds, &c. is evident : for, when any number consists of integer and decimal figures, the setting of the decimal point one place forward is the same thing as multiplying the number by 10 ; two places forward, the same as multiplying the number by 100 ; and so on : as the tables themselves are calculated only from

1 to 9. But 1 multiplied by 10 produces 10, and 9 multiplied by 10 produces 90. And so on for all the intermediate units.

X. *A Recommendation of Hadley's Quadrant for surveying, especially the surveying of Harbours, together with a particular Application of it in some Cases of Pilotage. By the Rev. John Michell, B. D. F. R. S.*

Read Feb. 14, 1765. **T**HE use of Hadley's quadrant, as an instrument to take altitudes at sea, is already so well established, that it wants no farther recommendation; but there are several other purposes, to which it may be applied, with great advantage, which, though obvious enough, seem yet to be hardly sufficiently attended to. There is no instrument so well adapted to many kinds of surveying, either for exactness or conveniency, and particularly the last; but the surveying of harbours, or such sands, as lie within sight of land, may oftentimes be performed by it, not only with vastly more ease, but also with a much greater degree of precision, than can be hoped for by any other means, as it is the only instrument in use, in which neither the exactness of the observations, nor the ease with which they may be taken, are sensibly affected by the motion of a vessel: and hence a single observer, in a boat, may generally determine the situation of any place
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